

MATH 106 CALCULUS I FOR BIO. & SOC. SCI. FALL 2012

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Homework 4.

Please show all your work.

- (1) Use the intermediate value theorem to show that there exists some x in the interval $[0, \frac{\pi}{2}]$ for which

$$\cos(x) = \frac{2x}{\pi}.$$

- (2) Show that the polynomial $p(x) = x^4 - x^2 - 10x + 1$ has at least one root. (Recall that a root of a polynomial $p(x)$ is a number c for which $p(c) = 0$.)
- (3) Let $w(x) = \sqrt{x+1}$. Compute

$$\lim_{h \rightarrow 0} \frac{w(h+1) - w(1)}{h}.$$

- (4) (a) Use the limit definition of the derivative to find $g'(1)$, if g is the function defined by $g(x) = x^2 + 1$. Calculations using rules for derivatives will not be accepted for this part.
- (b) Use the information obtained in part (a) to find the equation of the tangent line to the graph of $g(x)$ at the point $(1, 2)$.
- (5) Consider the function $f(x)$ defined below

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

- (a) Show that f is continuous at $x = 0$.
- (b) Use the definition of the derivative to show that f is not differentiable at $x = 0$; that is, use the limit definition of $f'(0)$ and show that it does not exist.